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Mobility of a forced Brownian particle

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Abstract : We study the mobility of a underdamped Brownian particle in a washboard (inclined 'periodic') potential system with a similarly periodic space dependent friction coefficient. We find that depending on the phase difference between the potential function and the friction coefficient the average mobility of the particle differs with respect to the sign of the slope of the washboard potential. We discuss the possible implications of the result.

Keywords : Mobility, Langevin equation, hysteresis.

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1. Introduction

The motion of an underdamped particle is difficult to study analytically and time consuming numerically. However, once the problem is solved the solutions offer useful information. Recently, hysteresis loops (average mobility *versus* applied periodic field) have been calculated for such systems and statistics of trapped time durations in potential wells (and running times between two consecutive trappings) was calculated showing power law behaviour [1]. It has further been shown [2] that when the average slope of the potential is small the average mobility of the particle is enhanced upon periodic modulation of the average slope of the potential. And if the slope is beyond the above range of small values mobility gets slackened. The enhancement of mobility is analogous to the reduction of effective friction coefficient of the medium studied so well in Tribology [3,4]. The enhancement of mobility is more pronounced when the friction coefficient of the medium is space dependent. The results obtained were for the simplest case of periodic variation of friction coefficient with the same periodicity as that of the periodic potential but with a phase difference, ϕ. Such inhomogeneous media one comes across in many physical and biological systems, the semiconductor heterostructure being an example. In some situations the system inhomogeneity plays a crucial role in particle motion which cannot be obtained in homogeneous media. In the present work we investigate such effects of system inhomogeneity on particle motion.

We consider the average slope of the otherwise periodic potential to be very small close to zero. We further consider the phase difference ϕ, between the potential function and the similarly periodic friction coefficient, being not equal to zero or π. ϕ ($\neq 0,\pi$) breaks the symmetry of the system either about the position where the periodic potential peaks or has a trough. As a result, we find that the average mobility of the particle for the small slope F and that for the slope $-F$ differ in magnitude. The difference, of course, depends on $|F|$ as well as the phase difference, ϕ. These dependences are presented (Section 2) in this work and their implications on the nature of particle motion discussed (Section 3).

2. The model and numerical results

The motion of the particle is described by the under-damped Langevin equation [5,6]

$$m\frac{d^2x}{dt^2}+\gamma(x)\frac{dx}{dt}-V_0k\cos(kx)-F-\sqrt{(k_BT\gamma(x))}\hat{f}(t)=0, \quad (1)$$

where $\gamma(x) = \gamma_0(1-\lambda \sin(kx + \phi))$ is the space dependent friction coefficient and $V(x) = - V_0 \sin(kx)$ is the potential profile. The external force F gives the average tilt to the potential. The fluctuating term $\hat{f}(t)$ satisfies $\langle \hat{f}(t)\rangle = 0$ and $\langle \hat{f}(t)\hat{f}(t')\rangle = 2\delta(t-t')$ and may represent thermal noise at temperature T. Eq. (1) is exactly the same as the Josephson junction equation except that one needs to replace ϕ by $\pi/2$ [7]. In our calculations we take $\lambda = 0.9$ (incidentally, the choice happens to be in conformity with the experimental value [7]) and for comparison $\lambda = 0$ (corresponding to uniform friction).

We solve the Langevin equation (1) (in fact, in its dimensionless form) numerically to calculate the average mobility $\mu| = \lim_{t\to\infty}\left(\frac{x(t)}{t}\right)/F\ |$ as a function of F. Here, $x(t)$ is the distance travelled by the particle in time t. We take $\frac{k_BT}{V_0} = 0.4$ implying that the noise strength is quite small compared to the potential barrier height ($\approx 2V_0$).

We numerically solve the Langevin equation [8,9] and calculate the displacement after a long time so that the relative uncertainty in the calculation of average reduced mobility is quite small, typically less than 0.025. We repeat the calculation for various small values of F and $-F$. The results of our calculation are shown in Figure 1. For the given value of the phase difference $\phi = 0.2$ (in terms of 2π) the mobilities for $+F$ are consistently smaller than those for $-F$. The result indicates that the particle is more mobile when the average slope of the potential is $+F$ than when the slope is $-F$. This implies that in the former case the medium offers less effective resistance to motion of the particle than in the latter case. This asymmetry in the behaviour of the motion of the particle is only because the medium is taken to be inhomogeneous; the medium appears different when seen either from the peak or the trough of the periodic potential (for $\phi \neq 0, \pi$). Furthermore, for $F = 0.05V_0k$ we evaluate the mobilities for various values of ϕ the results of which are presented in Figure 2. Interestingly, but perhaps not surprisingly, the difference of mobilities for $+F$ and $-F$ change sign as we increase ϕ from 0 to 0.5 ($0.5 < \phi < 1$ will just be the repetition of these calculations). It is to be noticed that the values of the mobilities themselves are very small. However, that the mobilities differ and the differences do not lie within the possible numerical errors is quite unmistakable as can typically be seen from Figure 3. These results suggest possible direction for further research.

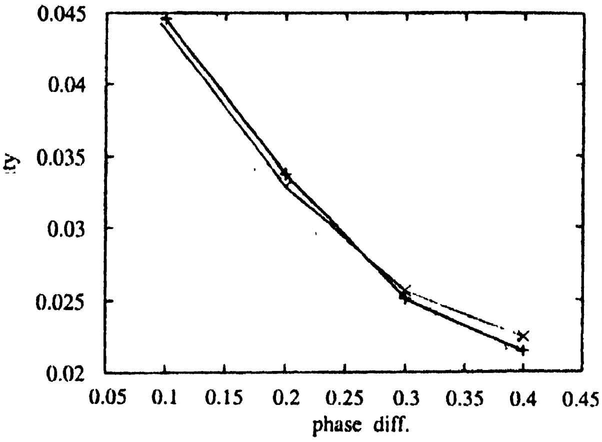

Figure 2. $\gamma_0\mu$ is plotted as a function of the phase difference ϕ for $F = \pm 0.05V_0k$. (Thick line for $F = 0.05V_0k$).

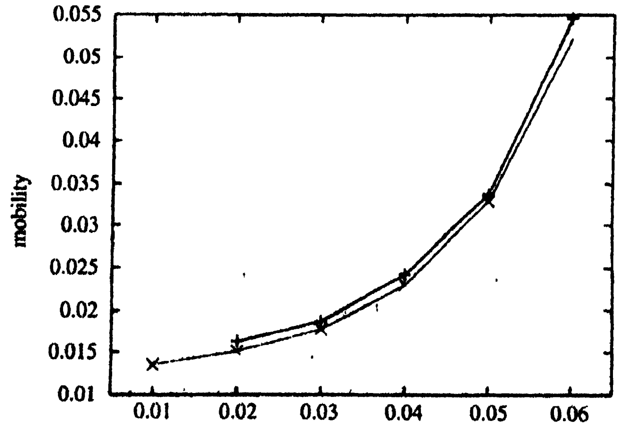

Figure 1. Shows the variation of $\gamma_0\mu$ ($\gamma_0 = 0.035\sqrt{(\mu V_0k^2)}$) as a function of tilt ($\pm F$ in units of V_0k). The data points joined by thick line correspond to $+F$. For this figure, the phase difference $\phi = 0.4\pi$.

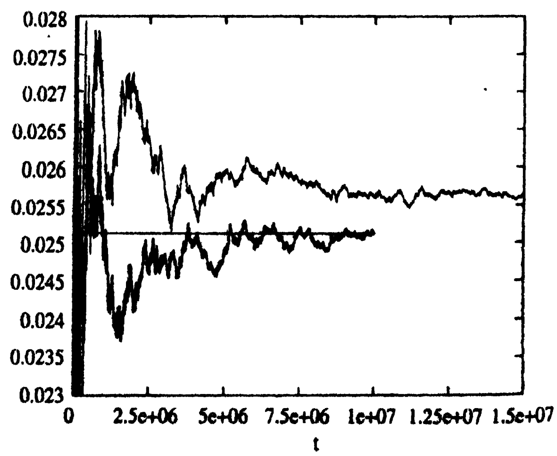

Figure 3. plots $\gamma_0\mu$ evaluated at time t (in units of $m^{-5}V_0^{-5}k^{-1}$) for $F = \pm 0.05V_0k$ with $\phi = 0.6\pi$(Thick line : $F = 0.05V_0k$). For large t the two curves are clearly separated from one another.

3. Summary and discussion

As mentioned in the introduction, for small slopes periodic modulation of the average slope of the potential results in the enhancement of mobility [2]. The present work indicates that the mobilites are different for very small slopes $+F$ and $-F$ and the difference depends on the average slope $|F|$ and the phase difference ϕ. These results clearly indicate that, for given values of ϕ ($\neq 0,\pi$) if the slope is periodically modulated around the average slope $F = 0$, we must get finite non-zero mobility. The implication is that even though the average force per period of modulation is equal to zero it is possible to get finite current even when the potential is perfectly periodic. This is possible only because the system inhomogeneity breaks the symmetry with respect to the direction of possible motion. Here, the presence of fluctuating forces (or thermal noise) is an essential ingredient for the particle motion. This indicates that thermal ratchet action (or net unidirectional current in an apparently unbiased periodic potential system), so well studied in overdamped systems [10], is possible even in underdamped systems. Overdamped systems simulate molecular motor movement in biological systems in the Brownian regime. The present work simulates mechanical motion in the inertial regime with analogies found [2,7] in Josephson junctions. The detailed exploration of the parameter space (F,ϕ) is expected to provide further insight into the nature of particle motion in such systems.

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References

[1] B Borromeo, G Constantini and F Marchesoni *Phys. Rev. Lett.* **82** 2820 (1999)

[2] M C Mahato and A M Jayannavar *Physica* **A318** 154 (2003)

[3] S Granick *Phys. Today* **52** 26 (1999)

[4] J Gao, W D Luedke and U Landmann *J. Phys. Chem.* **102** 5033 (1998); M Heuberger, C Drummond and J Israelachvili *ibid* **102** 5038 (1998)

[5] A M Jayannavar and M C Mahato *Pramana-J. Phys.* **45** 369 (1995); M C Mahato, T P Pareek and A M Jayannavar *Int. J. Mod. Phys.* **B10** 3857 (1996)

[6] V Ambegaokar and B I Halperin *Phys. Rev. Lett.* **22** 1364 (1969)

[7] C M Faico *Am. J. Phys.* **44** 733 (1976)

[8] W H Press, S A Teukolsky, W T Vetterling and B P Flannery *Numerical Recipes (in Fortran) : the Art of Scientific Computing* (Cambridge : Cambridge University Press) (1992)

[9] M C Mahato and S R Shenoy *J. Stat. Phys.* **73** 123 (1993)

[10] P Reimann *Phys. Rep.* **361** 57 (2002)